JIM 101: CALCULUS

Complex Numbers & Functions

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WEBEX 2

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2.00 PM-3.00 PM

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WEBEX CLASS 2 AGENDA

JIM101 CALCULUS

02 Complex Numbers

- ✓ Equality of Complex Numbers
- ✓ Argand Diagram
- ✓ Modulus and Argument
- ✓ Polar Form of a Complex Number
- ✓ De Moivre's Theorem

03 Functions

- ✓ What is a function?
- ✓ Notation
- ✓ Domain and Range
- ✓ Inverse Function

Equality of Complex Numbers

Let $a, b, c, d \in R$ and $i = \sqrt{-1}$. Any two complex numbers, $a + bi$ and $c + di$ are considered equal if and if both real part and the imaginary part are equal that is:

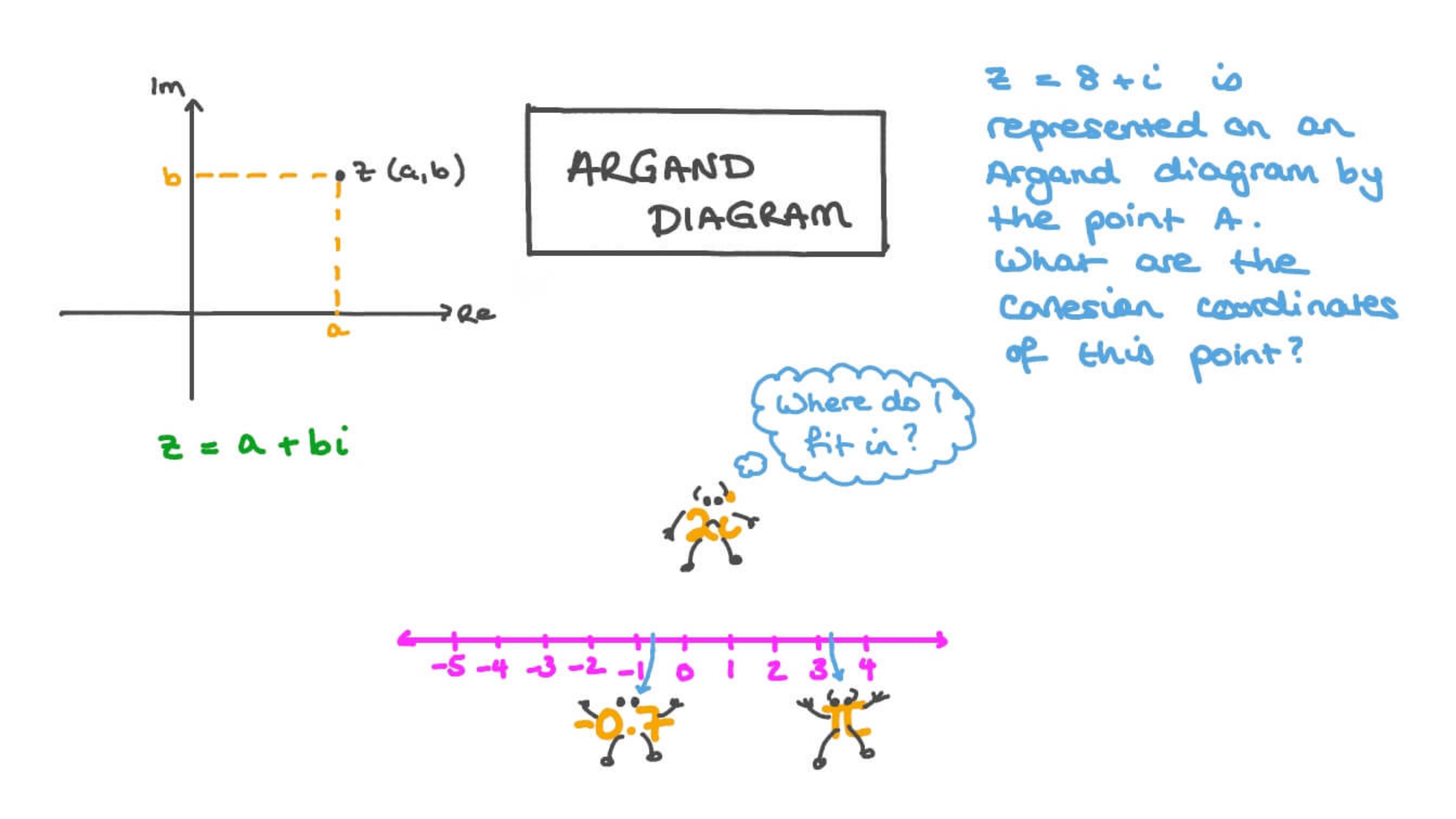
$$a + bi = c + di$$

Example 1

Solve the equation $(x + yi)(3 - i) = 1 + 2i$ where *x* and *y* are real numbers.

ARGAND DIAGRAM

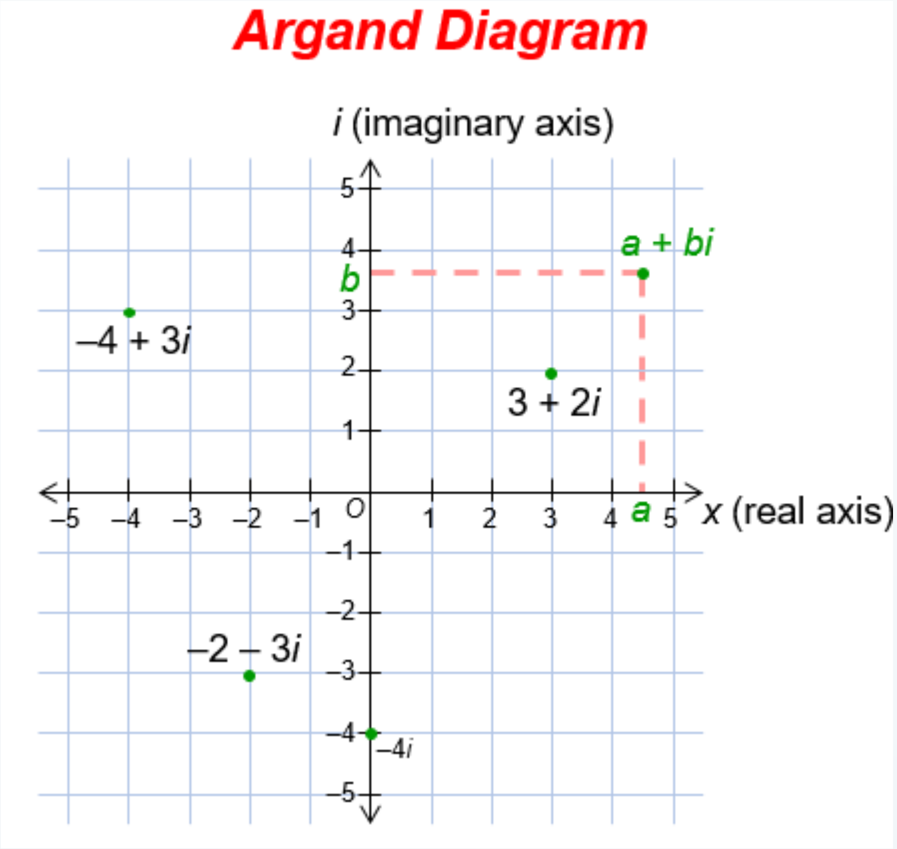
If Real numbers can be represented by points on straight line, then how do we represent Complex numbers?



ARGAND DIAGRAM

Complex numbers can be shown geometrically on an Argand diagram.

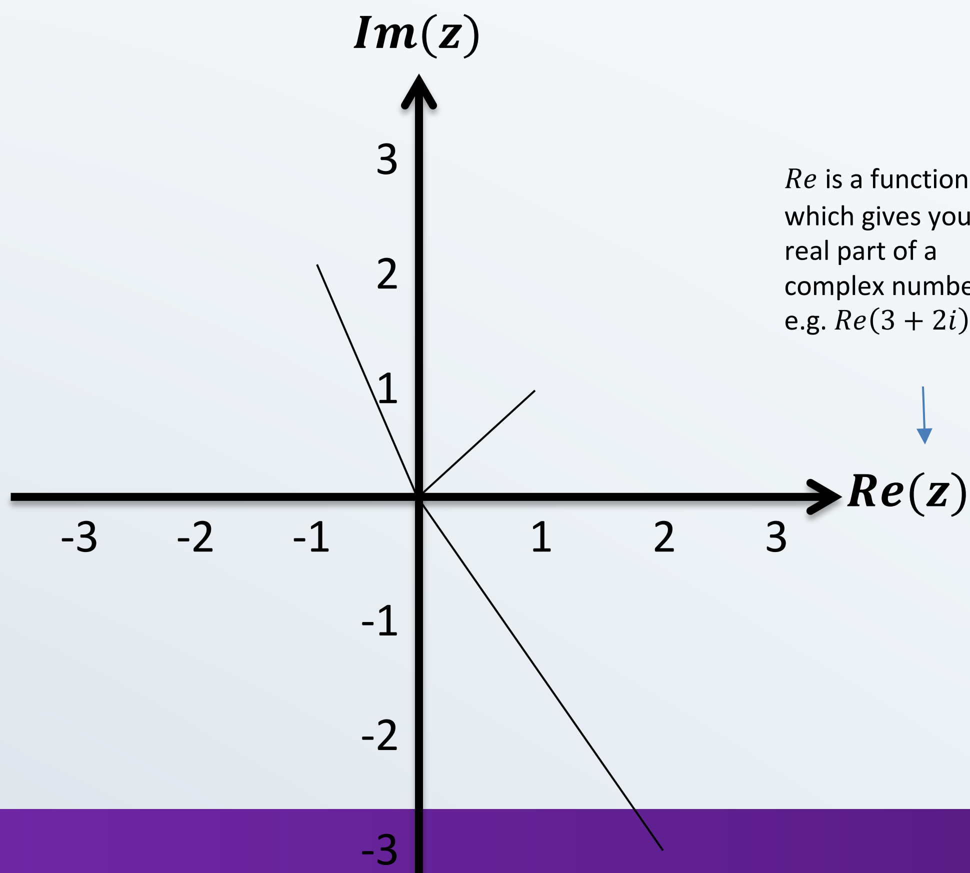
- *x-axis (Real axis)*
- *y-axis (Imaginary axis)*



ARGAND DIAGRAM

Just as x-y axes were a useful way to visualise coordinates, an Argand diagram allows us to visualise complex numbers.

Very simply, a complex number $x + iy$ can be plotted as a point (x, y).
The "x" axis is therefore the "**real axis**" and the "y" axis therefore the "**imaginary axis**".
The plane (2D space) formed by the axes is known as the "**complex plane**" or "z plane".



Re is a function which gives you the real part of a complex number.
e.g. $Re(3 + 2i) = 3$.

Click to move.

$1 + i$

$2 - 3i$

$-1 + 2i$

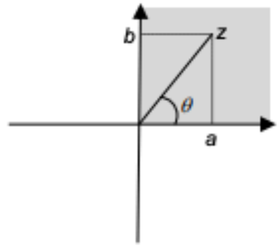
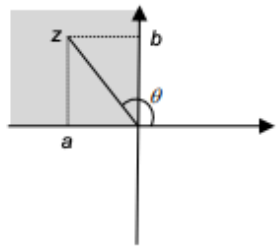
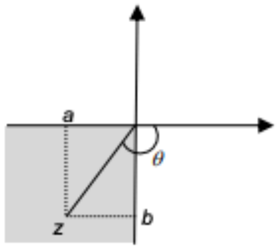
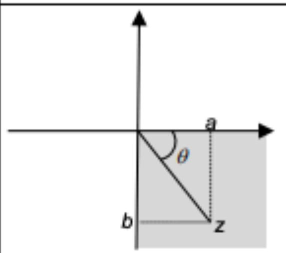
$-2i$

Magnitude or Modulus of z

- Let $z = x + yi$ be a complex number. The magnitude or modulus of z, denoted by $|z|$ is defined as the distance from the origin to the point (x, y). In other words:

$$|z| = \sqrt{x^2 + y^2}$$

ARGUMENT

Quadrant (shaded)	Argand diagram sketch	Argument: Information	Argument: Formula
1		*a* is positive *b* is positive argument is positive	$\theta = \tan^{-1}\left(\frac{b}{a}\right)$
2		*a* is negative *b* is positive argument is positive	$\theta = \pi + \tan^{-1}\left(\frac{b}{a}\right)$
3		*a* is negative *b* is negative argument is negative	$\theta = -\pi + \tan^{-1}\left(\frac{b}{a}\right)$
4		*a* is positive *b* is negative argument is negative	$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

Polar Form of a Complex Number

Convert Complex Number from Rectangular Form to Polar (Trigonometric) Form

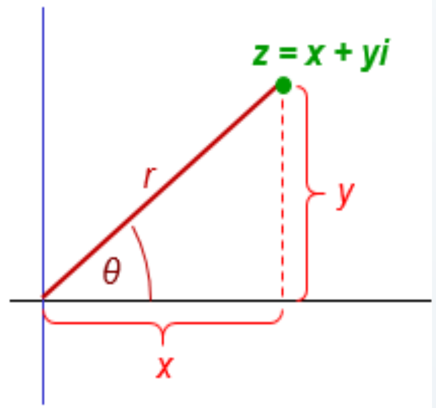
$z = x + yi$ (rectangular form)

$r = |z| = \sqrt{x^2 + y^2}$

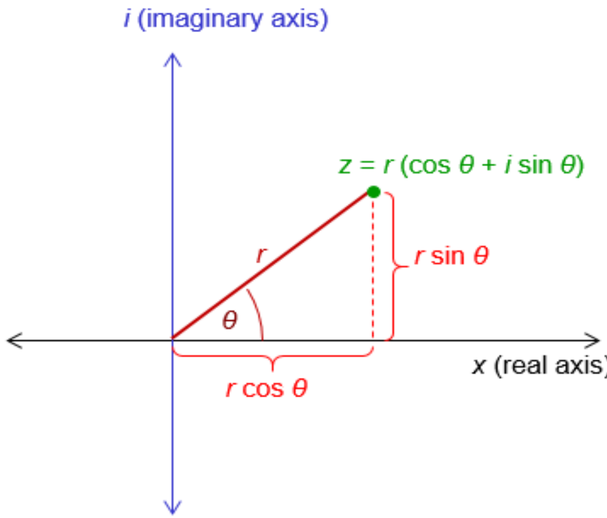
$x = r\cos\theta$

$y = r\sin\theta$

$z = r(\cos\theta + i\sin\theta)$ (polar form)



Complex Numbers In Polar Form



Example 2

Find the **modulus** and **argument** of the following complex number. Then, sketch the argand diagram.

$$z = 2 + i$$

Example 3

Find the **modulus** and **argument** of the following complex number. Then, sketch the argand diagram.

$$z = 5 - 4i$$

Example 4

Find the **modulus** and **argument** of the following complex number. Then, sketch the argand diagram.

$$z = -4 + 2i$$

Example 5

Find the **modulus** and **argument** of the following complex number. Then, sketch the argand diagram.

$$z = -1 - \sqrt{3}i$$

Example 6

Express the following in the **polar form**. Then, sketch the argand diagram.

$$z = 6 + 8i$$

Example 7

Express the following in the **polar form**. Then, sketch the argand diagram.

$$z = \frac{-1+5i}{2+3i}$$

De Moivre's Theorem

What is the fastest way to raise a Complex number to a specific power?

De Moivre's Theorem

Let $z = r(\cos\theta + i\sin\theta)$ and let n be a positive integer.

Then: $$z^n = [r(\cos\theta + i\sin\theta)]^n$$
$$= r^n(\cos n\theta + i\sin n\theta)$$

Example 8

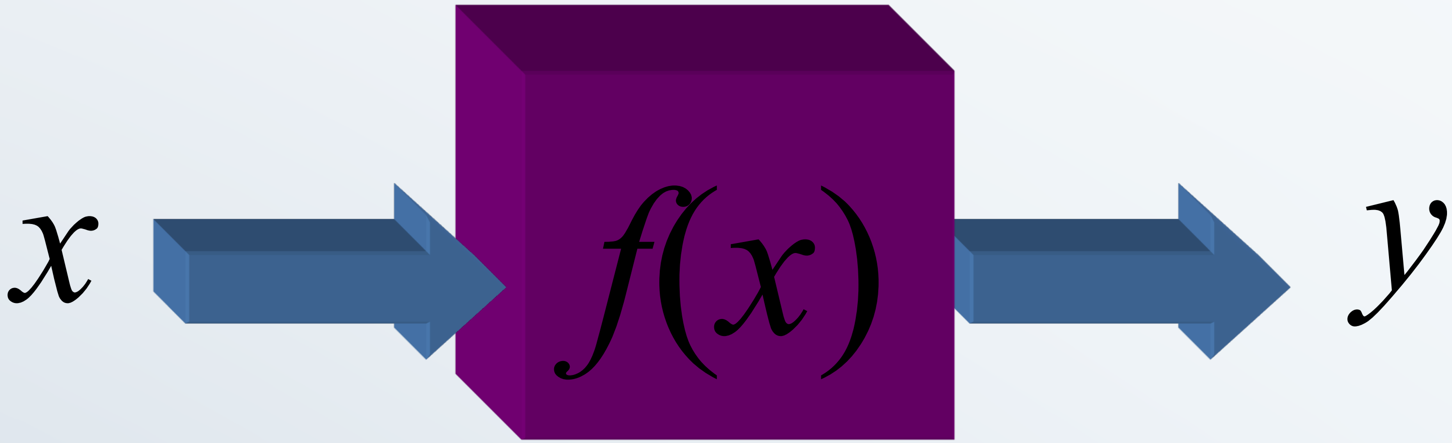
Find the value of $[2(\cos 60^{\circ} + i \sin 60^{\circ})]^2$

Example 9

Use DeMoivre's Theorem to find the 5th power of the complex number $z = 2(\cos 24^\circ + i \sin 24^\circ)$. Express the answer in the form $a + bi$.

Definition of a Function

A **<u>function</u>** is a relation in which each element of the domain is paired with *<u>exactly one</u>* element of the range. Another way of saying it is that there is <u>one and only one</u> output (*y*) with each input (*x*).



Notation of a Function

$$y = f(x)$$

Output

Name of Function

Input

Domain and Range of a Function

- A function is a set of ordered pairs of numbers (x, y) such that no x–values are repeated.

What are the domain and range of a function?

- The Domain is the set of all possible *x*-values in a function.
- The Range is the set of all possible *y*-values in a function.

Example 10

Given $f(x) = 8x - 1$, find:

(a) $f(2)$

(b) $f(1 - x)$

(c) the value of $k\ if\ f(k) = 3$

Example 11

Find the domain and range of $f(x) = 2x + 5$ by using the **graphical method.**

Example 12

Find the domain and range of $f(x) = |x + 1|$ by using the **graphical method.**

Example 13

Find the domain and range of $f(x) = \frac{5}{x^2}$ by using the **graphical method.**

Example 14

Find the domain and range of $f(x) = x^2 + 2$.

SOLUTION:

Since y defined for all values of x. Thus, the domain of the function:

$$D_f = (-\infty, \infty).$$

In order to find the range, we need to find the value of x.

$$x^2 + 2 = y$$
$$x^2 = y - 2$$
$$x = \pm\sqrt{y-2}$$

x is defined for values of y such that $\sqrt{y-2} \geq 0$, that is, $y \geq 2$.
Hence, the range of the function:

$$R_f = [2, \infty)$$

Inverse Function

If f is one-to-one, then f^{-1} exists.

The domain of f is the range of f^{-1}, and the range of f is the domain of f^{-1}.

If the point (a,b) is on the graph of f, then the point (b,a) is on the graph of f^{-1}, so the graphs of f and f^{-1} are reflections of each other across the line $y = x$.

Example 15

Find the inverse function of $f(x) = \frac{1}{x-3}$

Extra Exercises

1. Represent the following complex number on separate **Argand Diagrams**. Find the **modulus** and **argument** of the following complex number. Hence, express in **polar form**.

(a) $z_1 = 4 + 3i$
(b) $z_2 = -12 + 5i$
(c) $z_3 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$
(d) $z_4 = -8 - 3i$
(e) $z_5 = 3i$
(f) $z_6 = \frac{6+8i}{3-4i}$

Extra Exercises

2. Find the domain and range of the following functions.

(a) $f(x) = x^2 + 4x + 3$

(b) $f(x) = \frac{6}{x+2}$

(c) $f(x) = \sqrt{x - 6}$

(d) $f(x) = |x - 3|$

(e) $f(x) = -x^3 + 5$

ANNOUNCEMENT

- Kindly send me an email via asyrafman@usm.my for any inquires or to set up an appointment for JIM319 consultation.
- **Assignment 1 will be available to access from e-portal on 11th October 2019.**
- Do watch the pre-recorded e-kuliah or videos before attending WEBEX class for a better understanding.

NEXT WEBEX CLASS

LIMITS AND CONTINUITY

Kami Memimpin *We Lead*

THANK YOU SO MUCH

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