Different types of fuzzy sets

1. Type-2 fuzzy sets

Definition 1. A type-2 fuzzy set is a fuzzy set whose membership function values are type-1 fuzzy set (What we had earlier).

1. Interval-valued fuzzy sets

An interval-valued fuzzy set (IVFS) is a special case of type-2 fuzzy set. An IVFS is defined by a mapping F from the universe U to the set of closed intervals in $[0,1]$. Let $F(u) = [F_\star(u), F^\star(u)]$. The union, intersection and complementation of IVFSs are obtained by canonically extending fuzzy set-theoretic operations to intervals.

1. Type-m fuzzy sets

Definition 2. A type-m fuzzy set is a fuzzy set in X in whose membership values are type-$(m-1)$, $m > 1$ fuzzy sets on $[0,1]$.

1. Intuitionistic fuzzy sets

Given an underlying set X of objects, an intuitionistic fuzzy set (IFS) A is a set of ordered triples,

$$A = \{(x, \mu_A(x), \nu_A(x)|x \in X\}$$

where $\mu_A(x)$ and $\nu_A(x)$ are functions mapping from X into $[0,1]$. For each, $x \in X$, $\mu_A(x)$ represents the degree of membership functions of the element x to the subset A of X and $\nu_A(x)$ gives the degree of nonmembership. For the function $\mu_A(x)$ and $\nu_A(x)$ mapping into $[0,1]$, the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ holds.

Note. Ordinary sets over X may be viewed as special intuitionistic fuzzy sets with the nonmembership function $\nu_A(x) = 1 - \mu_A(x)$. In the same way as fuzzy sets, intuitionistic L-fuzzy sets were defined by mapping the membership functions into a partially ordered set L.

Extension of intuitionistic fuzzy sets

i. Pythagorean fuzzy number or set

Try to look for this on your own. Consider this as homework.

1. Fuzzy multisets

Let X be a nonempty set. A fuzzy multiset $\widetilde{A}$ drawn from X is characterized by a function, "count membership" of $\widetilde{A}$ denoted by CM_A such that $CM_A : X \to Q$ where Q is the set of all crisp multisets drawn from the unit interval $[0,1]$. Then for any $x \in X$, the value $CM_A(x)$ is a crisp multiset drawn from $[0,1]$. For each $x \in X$, the membership sequence is defined as the decreasingly ordered sequence of elements in $CM_A(x)$. It is denoted by $(\mu^1_{\widetilde{A}}(x), \mu^2_{\widetilde{A}}(x), \ldots, \mu^n_{\widetilde{A}}(x))$, where $\mu^1_{\widetilde{A}}(x) \geq \mu^2_{\widetilde{A}}(x) \geq \ldots \geq \mu^n_{\widetilde{A}}(x)$.

1. Hesitant fuzzy sets

Hesitant fuzzy sets (HFSs), are the extensions of regular fuzzy sets which handle the situations where a set of values are possible for the membership of a single element. Torra [175] defines hesitant fuzzy sets (HFSs) as follow:
Let X be a fixed set, a HFS on X is in terms of a function that when applied to X returns a subset of $[0, 1]$. Mathematical expression for HFS is as follows:

$$E = \{(x, h_E(x))|x \in X\}$$

where $h_E(x)$ is a set of some values in $[0, 1]$, denoting the possible membership degrees of the element $x \in X$ to the set E.