

Axioms for Intersection and Union

1. Intersection

$$I : [0, 1] \times [0, 1] \rightarrow [0, 1]$$

$$\mu_{\tilde{A} \cap \tilde{B}}(x) = I[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)]$$

Axiom 1: (Boundary condition)

$$I(1, 1) = 1, I(1, 0) = 0, I(0, 1) = 0, I(0, 0) = 0$$

Axiom 2: (Commutativity)

$$I(a, b) = I(b, a)$$

Axiom 3: (Monotonicity)

$$\text{If } b \leq c \text{ then } I(a, b) \leq I(a, c)$$

Axiom 4: (Associativity)

$$I(I(a, b), c) = I(a, I(b, c))$$

Axiom 5: (Continuity) I is a continuous function

Axiom 6: (Subidempotency)

$$I(a, a) \leq a$$

Axiom 7: (Strict Monotonicity)

$$I(a, b) < I(c, d) \text{ if } a < c \text{ and } b < d$$

1. Union

$$U : [0, 1] \times [0, 1] \rightarrow [0, 1]$$

$$\mu_{\tilde{A} \cup \tilde{B}}(x) = U[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)]$$

Axiom 1: (Boundary condition)

$$U(0, 0) = 0, U(0, 1) = 1, U(1, 0) = 1, U(1, 1) = 1$$

Axiom 2: (Commutativity)

$$U(a, b) = U(b, a)$$

Axiom 3: (Monotonicity)

$$\text{If } b \leq c \text{ then } U(a, b) \leq U(a, c)$$

Axiom 4: (Associativity)

$$U(U(a, b), c) = U(a, U(b, c))$$

Axiom 5: (Continuity) U is a continuous function

Axiom 6: (Subidempotency)

$$U(a, a) \geq a$$

Axiom 7: (Strict Monotonicity)

$$U(a, b) < U(c, d) \text{ if } a < c \text{ and } b < d$$