

VISUALISATION AND REPRESENTATION IN MATHEMATICS

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INTRODUCTION

- ▶ Visualization and representation is commonly believed to play an important role in mathematics teaching.
- ▶ visualization is a central component of many processes and is essential to fulfilling the transformation of thinking from the concrete to the abstract.
- ▶ The advantage of visualization is that it develops the power of multidimensional thinking in individuals. In this sense, individuals develop the capacity for collective discussion and exchange of ideas by looking at events from different perspectives.

VISUALISATION IN MATHEMATICS

- ▶ Visualization is a complicated process of transforming construction, mental images, and representations.
- ▶ It involves establishing a connection between information about ideas, which are previously unknown, and understandings, which gradually develop.
- ▶ Furthermore, it is a process, ability, and product of the reflection, usage, interpretation, and creation of pictures, images, and diagrams in our minds, on paper, or with technological tools for the purpose of portraying (Bishop, 2003; Hershkowitz, 1989; Wheatley, 1998; Zimmermann & Cunningham, 1991).

VISUALISATION

- ▶ Visualization in mathematics is the process of forming images (mentally, using pencil and paper or by using some technology) and effective implementation and use of these images for mathematical discovery and understanding of some mathematical problem.
- ▶ The researchers usually discuss the mathematical visualization in a figurative sense as "seeing the invisible" . Visual justification in mathematics refers to the understanding and application of mathematical concepts using visually based representations and processes presented in diagrams, computer graphics programs and physical models.
- ▶ The visualization as an approach in the teaching process is not only limited representation of drawings in order to illustrate certain objects or concepts, but it is used in every step of solving mathematical problems,

What Can Visualization Be Used For?

Purpose of visualising:

□ three purposes for visualising:

- ✓ to step into a problem,
- ✓ to model,
- ✓ to plan ahead.

1) Visualising used to step into a problem

- ▶ visualization can be applied to understanding the problem. Many students, who fail at solving word problems, also lack the ability to imagine the situation described in the word problem. Because they cannot visualize the problem correctly, they are not able to choose the right means to solve the problems.
- ▶ Here visualisations are used to help with understanding what the problem is about.
- ▶ The visualisation gives pupils the space to go deep into the situation to clarify and support their understanding before any generalisation can happen.

2) Visualising used to model

- ▶ visualization is the core technique used for modeling.
- ▶ This is particularly true for geometry - complex word problems often require the ability to imagine 3D or two-dimensional figures and relations between them, but this can be very difficult for students, who did not master visual modeling and are not able to use their mental images of figures to find the correct answer to the problem.

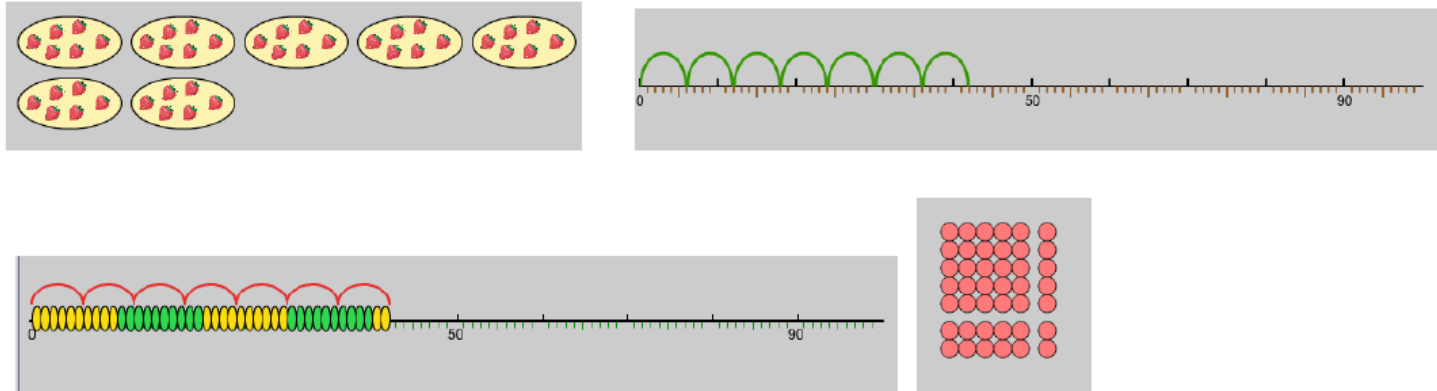
3) Visualising used to plan a head

- ▶ visualization is a powerful tool applied to planning ahead and making predictions about the future.
- ▶ This skill is also important for solving math problems, especially the ones, which require multiple operations on the equations performed in a correct order.
- ▶ Students, who are able to use visualization can predict what they will achieve if they apply certain rules of math. They do not need to test every possibility in practice, instead they can choose the correct procedure by evaluating the available options in their minds and predicting the possible outcomes.

REPRESENTATION IN MATHEMATICS

- ▶ **Representations** are useful in all areas of **mathematics** because they help us develop, share, and preserve our **mathematical** thoughts.
- ▶ "[They] help to portray, clarify, or extend a **mathematical** idea by focusing on its essential features" (NCTM, 2000, p. 206)
- ▶ Ball et al. (2008) also highlighted representations as being part of the 'specialised content knowledge' of mathematics unique to teaching. This specialised knowledge included selecting representations for particular purposes, recognising what is involved in using a particular representation, and linking representations to underlying ideas and other representations.

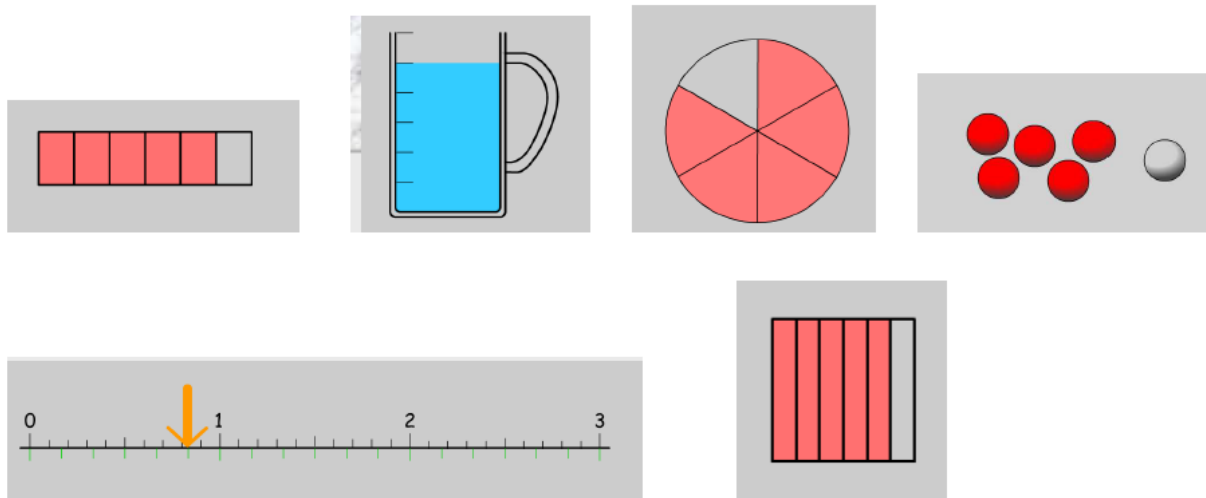
REPRESENTATION IN MATHEMATICS



Using these representations we can ask what characteristics of multiplication are emphasised by a particular representation and can consider the possibility of there being a key representation. Further we can then explore how the representations could be used to make sense of the various procedures that need to be understood.

REPRESENTATION IN MATHEMATICS

Similarly for exploring fractions the following representations were used:



Through exploring these representations and the relationship between them the pupils/trainees are encouraged to build up a language which facilitated a discussion about the nature/characteristics of fractions.

REPRESENTATION IN MATHEMATICS

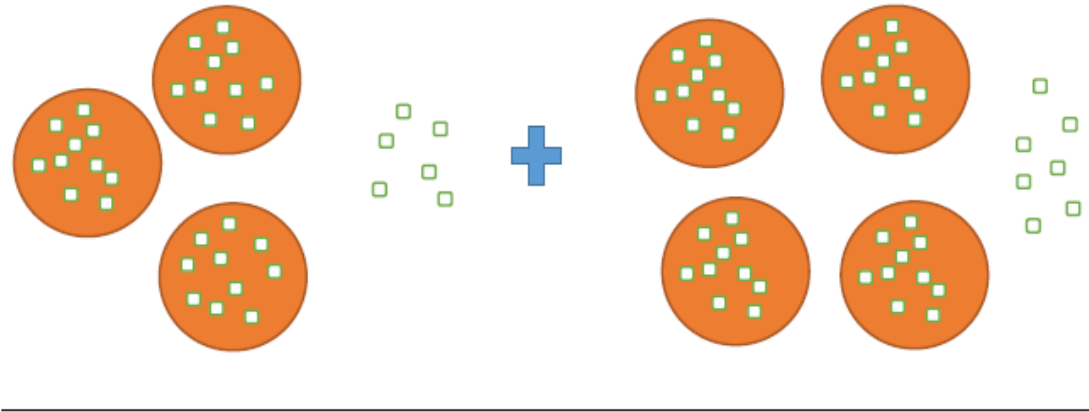


Table 1 – Focus Problems

Addition Problem	My brother went to a grocery store. He bought 8 apples and 7 oranges. How many pieces of fruit did my brother buy?
Subtraction Problem	Mr Smith has 13 computers in his store. He sold 5 computers. How many computers does Mr Smith have in his store now?
Subtraction Problem	Sally keeps 51 chicken. She sold 24 of the chickens at the market. Find the number of chicken that Sally has now?
Addition Problem	The Columbus Zoo has 36 seals and 47 penguins. Find the number of animals in the seal and penguin pools at the zoo.
Addition Problem	123 people visit the Butterfly exhibit on Monday and 98 people visit the Butterfly exhibit on Wednesday. How many people visit the Butterfly exhibit on both days?
Subtraction Problem	Alina bought a pair of shoes and a bag for 143 dollars. If Alina had 200 dollars to pay for these, how much change will she receive?

Example: The importance of visualization in the learning of integral calculus.

Problem 1:

Draw a figure to illustrate the geometric definition of the integral.

The geometric definition refers to the area concept

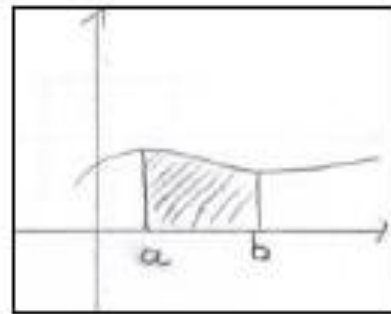


Figure 1

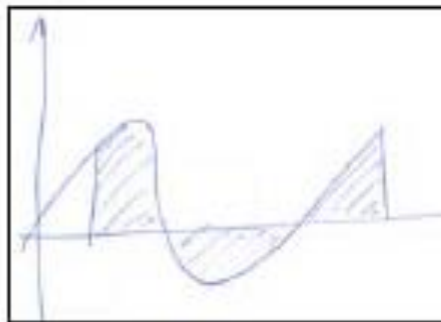
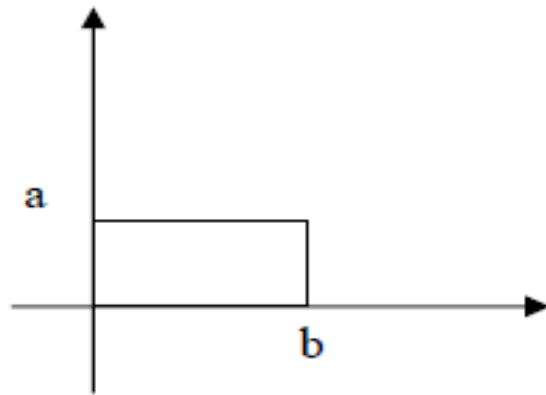


Figure 2

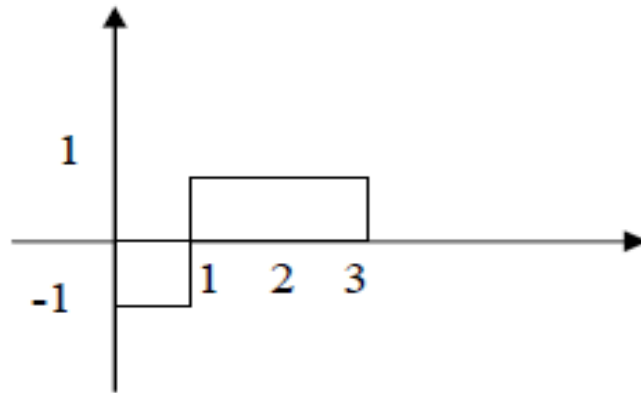
Problem 2:

Find a formula for the area by using integration.

a)



b)



Mathematics is cognitive process-thinking-that requires the dual coding of imagery and language.

Discuss the relationship between the visualisation and representation in mathematics from the perspective of Cognitive learning theories.

summary

The term 'visualization' is used to refer to both internal mental representations and external displays. Internal spatial visualisation ability can be interpreted as the ability to mentally store and manipulate visual-spatial representations in the mind (Hegarty & Waller, 2005). External visualisations are visual-spatial displays that occur in the world and comprise both static images, such as drawings, graphs, charts and diagrams, and dynamic representations, such as animations. Both forms of visualisation play important roles in scientific and mathematical problem solving (Ferguson, 1977; Miller, 1986).

summary

- ▶ Common **examples of mental images** include daydreaming and the **mental visualization** that occurs while reading a book.
- ▶ Another is of the pictures summoned by athletes during training or before a competition, outlining each step they will take to accomplish their goal.